# EUF

# Exame Unificado das Pós-graduações em Física

Para o primeiro semestre de 2015

14-15 outubro 2014

# FORMULÁRIO

Não escreva nada neste formulário. Devolva-o ao fim do primeiro dia de prova.

# Constantes físicas

| | |
|---|---|
| Velocidade da luz no vácuo | $c = 3{,}00\times10^{8}$ m/s |
| Constante de Planck | $h = 6{,}63\times10^{-34}$ J s $= 4{,}14\times10^{-15}$ eV s |
| | $hc = 1240$ eV nm |
| Constante de Wien | $W = 2{,}898\times10^{-3}$ m K |
| Permeabilidade magnética do vácuo | $\mu_0 = 4\pi\times10^{-7}$ N/A$^2$ $= 12{,}6\times10^{-7}$ N/A$^2$ |
| Permissividade elétrica do vácuo | $\epsilon_0 = \dfrac{1}{\mu_0 c^2} = 8{,}85\times10^{-12}$ F/m |
| | $\dfrac{1}{4\pi\epsilon_0} = 8{,}99\times10^{9}$ Nm$^2$/C$^2$ |
| Constante gravitacional | $G = 6{,}67\times10^{-11}$ N m$^2$/kg$^2$ |
| Carga elementar | $e = 1{,}60\times10^{-19}$ C |
| Massa do elétron | $m_{\rm e} = 9{,}11\times10^{-31}$ kg $= 511$ keV/c$^2$ |
| Comprimento de onda Compton | $\lambda_{\rm C} = 2{,}43\times10^{-12}$ m |
| Massa do próton | $m_{\rm p} = 1{,}673\times10^{-27}$ kg $= 938$ MeV/c$^2$ |
| Massa do nêutron | $m_{\rm n} = 1{,}675\times10^{-27}$ kg $= 940$ MeV/c$^2$ |
| Massa do dêuteron | $m_{\rm d} = 3{,}344\times10^{-27}$ kg $= 1.876$ MeV/c$^2$ |
| Massa da partícula $\alpha$ | $m_\alpha = 6{,}645\times10^{-27}$ kg $= 3.727$ MeV/c$^2$ |
| Constante de Rydberg | $R_H = 1{,}10\times10^{7}$ m$^{-1}$ , $\quad R_H hc = 13{,}6$ eV |
| Raio de Bohr | $a_0 = 5{,}29\times10^{-11}$ m |
| Constante de Avogadro | $N_{\rm A} = 6{,}02\times10^{23}$ mol$^{-1}$ |
| Constante de Boltzmann | $k_{\rm B} = 1{,}38\times10^{-23}$ J/K $= 8{,}62\times10^{-5}$ eV/K |
| Constante universal dos gases | $R = 8{,}31$ J mol$^{-1}$ K$^{-1}$ |
| Constante de Stefan-Boltzmann | $\sigma = 5{,}67\times10^{-8}$ W m$^{-2}$ K$^{-4}$ |

| | | | | | |
|---|---|---|---|---|---|
| Raio do Sol | = | $6{,}96\times10^{8}$ m | Massa do Sol | = | $1{,}99\times10^{30}$ kg |
| Raio da Terra | = | $6{,}37\times10^{6}$ m | Massa da Terra | = | $5{,}98\times10^{24}$ kg |
| Distância Sol-Terra | = | $1{,}50\times10^{11}$ m | | | |

1 J $= 10^7$ erg $\qquad$ 1 eV $= 1{,}60\times10^{-19}$ J

# Constantes numéricas

| | | |
|---|---|---|
| $\pi \cong 3{,}142$ | $\ln 2 \cong 0{,}693$ | $\cos(30°) = \sin(60°) = \sqrt{3}/2 \cong 0{,}866$ |
| $\mathrm{e} \cong 2{,}718$ | $\ln 3 \cong 1{,}099$ | $\sin(30°) = \cos(60°) = 1/2$ |
| $1/\mathrm{e} \cong 0{,}368$ | $\ln 5 \cong 1{,}609$ | |
| $\log_{10}\mathrm{e} \cong 0{,}434$ | $\ln 10 \cong 2{,}303$ | |

# Mecânica Clássica

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \qquad \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \mathbf{r} \times \mathbf{F} \qquad L_i = \sum_j I_{ij}\omega_j \qquad T_R = \sum_{ij} \frac{1}{2} I_{ij}\omega_i\omega_j \qquad I = \int r^2 \, dm$$

$$\mathbf{r} = r\hat{\mathrm{e}}_r \qquad \mathbf{v} = \dot{r}\hat{\mathrm{e}}_r + r\dot{\theta}\hat{\mathrm{e}}_\theta \qquad \mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\mathrm{e}}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\mathrm{e}}_\theta$$

$$\mathbf{r} = \rho\hat{\mathrm{e}}_\rho + z\hat{\mathrm{e}}_z \qquad \mathbf{v} = \dot{\rho}\hat{\mathrm{e}}_\rho + \rho\dot{\varphi}\hat{\mathrm{e}}_\varphi + \dot{z}\hat{\mathrm{e}}_z \qquad \mathbf{a} = \left(\ddot{\rho} - \rho\dot{\varphi}^2\right)\hat{\mathrm{e}}_\rho + \left(\rho\ddot{\varphi} + 2\dot{\rho}\dot{\varphi}\right)\hat{\mathrm{e}}_\varphi + \ddot{z}\hat{\mathrm{e}}_z$$

$$\mathbf{r} = r\hat{\mathrm{e}}_r \qquad \begin{aligned}\mathbf{v} = {} & \dot{r}\hat{\mathrm{e}}_r + r\dot{\theta}\hat{\mathrm{e}}_\theta \\ & + r\dot{\varphi}\sin\theta\hat{\mathrm{e}}_\varphi\end{aligned} \qquad \begin{aligned}\mathbf{a} = {} & \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\varphi}^2\sin^2\theta\right)\hat{\mathrm{e}}_r \\ & + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\varphi}^2\sin\theta\cos\theta\right)\hat{\mathrm{e}}_\theta \\ & + \left(r\ddot{\varphi}\sin\theta + 2\dot{r}\dot{\varphi}\sin\theta + 2r\dot{\theta}\dot{\varphi}\cos\theta\right)\hat{\mathrm{e}}_\varphi\end{aligned}$$

$$E = \frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} + V(r) \qquad V(r) = -\int_{r_0}^{r} F(r')\mathrm{d}r' \qquad V_{\text{efetivo}} = \frac{L^2}{2mr^2} + V(r)$$

$$\int_{R_0}^{R} \frac{\mathrm{d}r}{\sqrt{E - V(r) - \frac{L}{2mr^2}}} = \sqrt{\frac{2}{m}}\,(t - t_0) \qquad \dot{\theta} = \frac{L}{mr^2}$$

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\theta^2} + u = -\frac{m}{L^2u^2}F\left(1/u\right), \quad u = \frac{1}{r}; \qquad \left(\frac{\mathrm{d}u}{\mathrm{d}\theta}\right)^2 + u^2 = \frac{2m}{L^2}\left[E - V(1/u)\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0, \quad L = T - V \qquad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k} = Q_k$$

$$Q_k = \sum_{i=1}^{N} F_{ix}\frac{\partial x_i}{\partial q_k} + F_{iy}\frac{\partial y_i}{\partial q_k} + F_{iz}\frac{\partial z_i}{\partial q_k} \qquad Q_k = -\frac{\partial V}{\partial q_k}$$

$$\left(\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\right)_{\text{rotação}} = \left(\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\right)_{\text{fixo}} - 2\boldsymbol{\omega} \times \mathbf{v}' - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - \dot{\boldsymbol{\omega}} \times \mathbf{r}$$

$$H = \sum_{k=1}^{f} p_k\dot{q}_k - L; \qquad \dot{q}_k = \frac{\partial H}{\partial p_k}; \qquad \dot{p}_k = -\frac{\partial H}{\partial q_k}; \qquad \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

# Eletromagnetismo

$$\oint \mathbf{E}\cdot \mathrm{d}\vec{\ell} + \frac{\partial}{\partial t}\int \mathbf{B}\cdot \mathrm{d}\mathbf{S} = 0 \qquad \nabla\times\mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\oint \mathbf{B}\cdot \mathrm{d}\mathbf{S} = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

$$\oint \mathbf{D}\cdot \mathrm{d}\mathbf{S} = Q = \int \rho \mathrm{d}V \qquad \nabla\cdot\mathbf{D} = \rho$$

$$\oint \mathbf{H}\cdot \mathrm{d}\vec{\ell} - \frac{\partial}{\partial t}\int \mathbf{D}\cdot \mathrm{d}\mathbf{S} = I = \int \mathbf{J}\cdot \mathrm{d}\mathbf{S} \qquad \nabla\times\mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$$

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon\mathbf{E} \qquad \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu\mathbf{H}$$

$$\oint \mathbf{P}\cdot \mathrm{d}\mathbf{S} = -Q_P \qquad \nabla\cdot\mathbf{P} = -\rho_P \qquad \oint \mathbf{M}\cdot \mathrm{d}\vec{\ell} = I_M \qquad \nabla\times\mathbf{M} = \mathbf{J}_M$$

$$V = -\int \mathbf{E}\cdot \mathrm{d}\vec{\ell} \qquad \mathbf{E} = -\nabla V \qquad \mathrm{d}\mathbf{H} = \frac{I\mathrm{d}\vec{\ell}\times\hat{\mathrm{e}}_r}{4\pi r^2} \qquad \mathbf{B} = \nabla\times\mathbf{A}$$

$$\mathrm{d}\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}\hat{\mathrm{e}}_r \qquad \mathrm{d}V = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r} \qquad \mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \qquad \mathrm{d}\mathbf{F} = I\mathrm{d}\vec{\ell}\times\mathbf{B}$$

$$\mathbf{J} = \sigma\mathbf{E} \qquad \nabla\cdot\mathbf{J} + \frac{\partial\rho}{\partial t} = 0$$

$$u = \frac{1}{2}\left(\mathbf{D}\cdot\mathbf{E} + \mathbf{B}\cdot\mathbf{H}\right) \qquad \mathbf{S} = \mathbf{E}\times\mathbf{H} \qquad \mathbf{A} = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}\mathrm{d}V}{r}$$

$$(\rho = 0, \mathbf{J} = \mathbf{0}) \Rightarrow \nabla^2\mathbf{E} = \mu\epsilon\frac{\partial^2\mathbf{E}}{\partial t^2} \qquad n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\hat{\mathrm{e}}_r \qquad U = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r} \qquad \mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathrm{e}}_r \qquad V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

# Relatividade

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \qquad x' = \gamma\,(x - Vt) \qquad t' = \gamma\left(t - Vx/c^2\right)$$

$$v'_x = \frac{v_x - V}{1 - Vv_x/c^2} \qquad v'_y = \frac{v_y}{\gamma\,(1 - Vv_x/c^2)} \qquad v'_z = \frac{v_z}{\gamma\,(1 - Vv_x/c^2)}$$

$$E = mc^2 = \gamma m_0c^2 = m_0c^2 + K \qquad E = \sqrt{(pc)^2 + (m_0c^2)^2}$$

# Mecânica Quântica

$$i\hbar\,\frac{\partial\Psi(x,t)}{\partial t} = H\Psi(x,t) \qquad H = \frac{-\hbar^2}{2m}\,\frac{1}{r}\,\frac{\partial^2}{\partial r^2}\,r + \frac{\hat{L}^2}{2mr^2} + V(r)$$

$$p_x = \frac{\hbar}{i}\frac{\partial}{\partial x} \qquad [x, p_x] = i\hbar$$

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + i\frac{\hat{p}}{m\omega}\right) \qquad \hat{a}|n\rangle = \sqrt{n}|n-1\rangle\ , \qquad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$L_\pm = L_x \pm iL_y \qquad L_\pm Y_{\ell m}(\theta,\varphi) = \hbar\sqrt{l(l+1) - m(m\pm 1)}\ Y_{\ell m\pm 1}(\theta,\varphi)$$

$$L_z = x\,p_y - y\,p_x \qquad L_z = \frac{\hbar}{i}\frac{\partial}{\partial\varphi}\ , \qquad [L_x, L_y] = i\hbar L_z$$

$$E_n^{(1)} = \langle n|\delta H|n\rangle \qquad E_n^{(2)} = \sum_{m\neq n}\frac{|\langle m|\delta H|n\rangle|^2}{E_n^{(0)} - E_m^{(0)}}\ , \qquad \phi_n^{(1)} = \sum_{m\neq n}\frac{\langle m|\delta H|n\rangle}{E_n^{(0)} - E_m^{(0)}}\phi_m^{(0)}$$

$$\hat{\mathbf{S}} = \frac{\hbar}{2}\vec{\sigma} \qquad \sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\ , \quad \sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\ , \quad \sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\bar{\psi}(\vec{p}) = \frac{1}{(2\pi\hbar)^{3/2}}\int d^3r\,e^{-i\vec{p}\cdot\vec{r}/\hbar}\,\psi(\vec{r}) \qquad \psi(\vec{r}) = \frac{1}{(2\pi\hbar)^{3/2}}\int d^3p\,e^{i\vec{p}\cdot\vec{r}/\hbar}\,\bar{\psi}(\vec{p})$$

# Física Moderna

$$p = \frac{h}{\lambda} \qquad E = h\nu = \frac{hc}{\lambda} \qquad E_n = -Z^2\,\frac{hcR_H}{n^2}$$

$$R_T = \sigma T^4 \qquad \lambda_{\max}T = b \qquad L = mvr = n\hbar$$

$$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta) \qquad n\lambda = 2d\sin\theta \qquad \Delta x\ \Delta p \geq \hbar/2$$

# Termodinâmica e Mecânica Estatística

$$dU = dQ - dW \qquad dU = TdS - pdV + \mu dN$$

$$dF = -SdT - pdV + \mu dN \qquad dH = TdS + Vdp + \mu dN$$

$$dG = -SdT + Vdp + \mu dN \qquad d\Phi = -SdT - pdV - Nd\mu$$

$$F = U - TS \qquad G = F + pV$$

$$H = U + pV \qquad \Phi = F - \mu N$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V \qquad \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p \qquad \left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_T \qquad S = -\left(\frac{\partial F}{\partial T}\right)_V$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V \qquad C_p = \left(\frac{\partial H}{\partial T}\right)_p = T\left(\frac{\partial S}{\partial T}\right)_p$$

Gás ideal: $pV = nRT$, $U = ncT$, $pV^\gamma = \text{const.}$, $\gamma = (c+R)/c$

$$S = k_B \ln W$$

$$Z = \sum_n e^{-\beta E_n} \qquad Z = \int d\gamma e^{-\beta E(\gamma)} \qquad \beta = 1/k_B T$$

$$F = -k_B T \ln Z \qquad U = -\frac{\partial}{\partial \beta} \ln Z$$

$$\Xi = \sum_N Z_N e^{\beta \mu N} \qquad \Phi = -k_B T \ln \Xi$$

$$f_{\text{FD}} = \frac{1}{e^{\beta(\epsilon-\mu)} + 1} \qquad f_{\text{BE}} = \frac{1}{e^{\beta(\epsilon-\mu)} - 1}$$

# Resultados matemáticos

$$\int_{-\infty}^{\infty} x^{2n} \mathrm{e}^{-\alpha x^2} \mathrm{d}x = \frac{1.3.5...(2n{+}1)}{(2n{+}1)2^n \alpha^n} \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}} \qquad \sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \quad (|x|<1)$$

$$\int \frac{\mathrm{d}u}{u(u-1)} = \ln(1-1/u) \qquad \mathrm{e}^{i\theta} = \cos\theta + i\sin\theta$$

$$\int \frac{\mathrm{d}z}{(a^2+z^2)^{1/2}} = \ln\left(z+\sqrt{z^2+a^2}\right) \qquad \ln N! \approx N\ln N - N$$

$$\int \frac{\mathrm{d}u}{1-u^2} = \frac{1}{2}\ln\left(\frac{1+u}{1-u}\right) \qquad \int_{-\infty}^{\infty} \exp(-\alpha t^2)\mathrm{d}t = \sqrt{\frac{\pi}{\alpha}}$$

$$\int \frac{1}{a^2+y^2}\,dy = \frac{1}{a}\ \arctan\frac{y}{a} \qquad \int \frac{x}{a^2+x^2}\,dx = \frac{1}{2}\ \ln(a^2+x^2)$$

$$\int_0^{\infty} \frac{z^{x-1}}{e^z+1}\,dz = (1-2^{1-x})\ \Gamma(x)\ \zeta(x) \qquad (x>0)$$

$$\int_0^{\infty} \frac{z^{x-1}}{e^z-1}\,dz = \Gamma(x)\ \zeta(x) \qquad (x>1)$$

$$\Gamma(2)=1 \qquad \Gamma(3)=2 \qquad \Gamma(4)=6 \qquad \Gamma(5)=24$$

$$\zeta(2) = \frac{\pi^2}{6} = 1{,}645 \qquad \zeta(3) = 1{,}202 \qquad \zeta(4) = \frac{\pi^4}{90} = 1{,}082 \qquad \zeta(5) = 1{,}037$$

$$\int_{-\pi}^{\pi} \sin(mx)\ \sin(nx)\ dx = \pi\delta_{m,n} \qquad \int_{-\pi}^{\pi} \cos(mx)\ \cos(nx)\ dx = \pi\delta_{m,n}$$

$$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \rho\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}z \qquad \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = r^2\mathrm{d}r\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

$$Y_{0,0} = \sqrt{\frac{1}{4\pi}} \qquad Y_{1,0} = \sqrt{\frac{3}{4\pi}}\cos\theta \qquad Y_{1,\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\mathrm{e}^{\pm i\phi}$$

$$Y_{2,0} = \sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta - 1\right) \qquad Y_{2,\pm1} = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\mathrm{e}^{\pm i\phi} \qquad Y_{2,\pm2} = \mp\sqrt{\frac{15}{32\pi}}\sin^2\theta\mathrm{e}^{\pm 2i\phi}$$

$$P_0(x) = 1 \qquad P_1(x) = x \qquad P_2(x) = (3x^2-1)/2$$

Solução geral para a equação de Laplace em coordenadas esféricas, com simetria azimutal:

$$V(r,\theta) = \sum_{l=0}^{\infty}(A_l r^l + \frac{B_l}{r^{l+1}})\ P_l(\cos\theta)$$

$$\oint \mathbf{A}\cdot \mathrm{d}\mathbf{S} = \int (\nabla\cdot\mathbf{A})\,\mathrm{d}V \qquad\qquad \oint \mathbf{A}\cdot \mathrm{d}\vec{\ell} = \int (\nabla\times\mathbf{A})\cdot\mathrm{d}\mathbf{S}$$

*Coordenadas cartesianas*

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathrm{e}}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathrm{e}}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathrm{e}}_z$$

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathrm{e}}_x + \frac{\partial f}{\partial y}\hat{\mathrm{e}}_y + \frac{\partial f}{\partial z}\hat{\mathrm{e}}_z \qquad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

*Coordenadas cilíndricas*

$$\nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right]\hat{\mathrm{e}}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right]\hat{\mathrm{e}}_\varphi + \left[\frac{1}{\rho}\frac{\partial(\rho A_\varphi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial \varphi}\right]\hat{\mathrm{e}}_z$$

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\mathrm{e}}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\hat{\mathrm{e}}_\varphi + \frac{\partial f}{\partial z}\hat{\mathrm{e}}_z \qquad \nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

*Coordenadas esféricas*

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial(A_\varphi)}{\partial \varphi}$$

$$\nabla\times\mathbf{A} = \left[\frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\varphi)}{\partial \theta} - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial \varphi}\right]\hat{\mathrm{e}}_r$$

$$+ \left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{1}{r}\frac{\partial(rA_\varphi)}{\partial r}\right]\hat{\mathrm{e}}_\theta + \left[\frac{1}{r}\frac{\partial(rA_\theta)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right]\hat{\mathrm{e}}_\varphi$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathrm{e}}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\mathrm{e}}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\mathrm{e}}_\varphi$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$$